

**Model Question Paper-01**

**Physical Sciences**  
**Test Booklet**

Sr. No \_\_\_\_\_  
Subject Code: 3

Time: 3:00 Hours

Maximum Marks: 200

**INSTRUCTIONS**

1. **You have opted for English as medium of Question Paper.** This Test Booklet contains one hundred and forty five (20 Part ‘A’ + 25 Part ‘B’ + 30 Part ‘C’) Multiple Choice Questions (MCQs). You are required to answer a maximum of 15, 20 and 20 questions form Part ‘A’ ‘B’ and ‘C’ respectively. If more than required number of questions are answered, only first 15, 20 and 20 questions in Part ‘A’ ‘B’ and ‘C’ respectively, will be taken up for evaluation.
2. Answer sheet has been provided separately. Before you start filling up your particulars, please ensure that the booklet contains requisite number of pages and that these are not torn or mutilated. If it is so, you may request the Invigilator to change the booklet. Likewise, check the answer sheet also. Sheets to rough work have been appended to the test booklet.
3. Write your Roll No., name, Your address and Serial Number of this Test booklet on the Answer sheet in the space provided on the side 1 of Answer sheet. Also put your signatures in the space identified.
4. **You must darken the appropriate circles with a pencil related to Roll Number, Subject Code, Booklet Code and Center Code on the OMR answer sheet. It is the sole responsibility of the candidate to meticulously follow the instructions given on Answer Sheet, failing which, the computer shall not be able to decipher the correct detail which may ultimately result in loss, including rejection of the OMR answer sheet.**
5. Each question in part ‘A’ carries 2 marks, Part ‘B’ 3.5 marks and Part ‘C’ 5 marks respectively. There will be negative marking @ 25% for each wrong answer.
6. Below each question in Part ‘A’, ‘B’ and ‘C’ four alternatives or responses are given. Only one of these alternatives is the “correct” option to the question. You have to find, for each question, the correct or the best answer.
7. Candidates found copying or resorting to any unfair means are liable to be disqualified from this and future examinations.
8. Candidates should not write anything anywhere except on answer sheet or sheets for rough work.
9. After the test is over, you MUST hand over the Test Booklet and the answer sheet (OMR) to the invigilator.
10. Use of calculator is not permitted.

Roll No. :.....

I have verified all the information filled  
in by the candidate

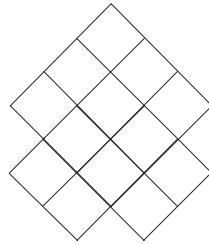
Name: .....

.....  
Signature of the Invigilator

**PART – A**

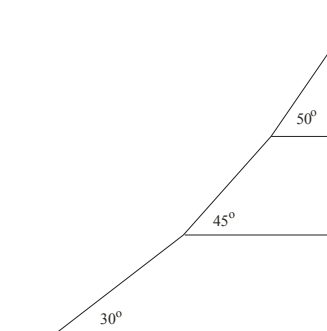
1. In still air, fragrance of a burning incense stick will be simply by an observer quickest when the experiment is carried out at
  1. low altitude and high air temperature
  2. high altitude and low air temperature
  3. low altitude and low air temperature
  4. high altitude and high air temperature

2. How many squares are there in this figure?



- |       |       |
|-------|-------|
| 1. 9  | 2. 14 |
| 3. 15 | 4. 17 |

3. A mountain road has 3 sections of different slopes as shown. What is the average slope  $m$  of the entire climb?



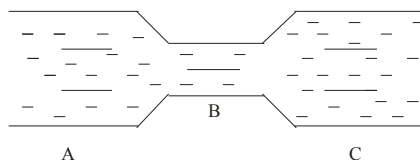
- |                       |                           |
|-----------------------|---------------------------|
| 1. 1                  | 2. $(1/3) < m < (1/2)$    |
| 3. $1 < m < \sqrt{3}$ | 4. $(1/\sqrt{3}) < m < 1$ |



1. A and B have equal weights.
  2. A is heavier than B by an amount  $w$ .
  3. A is heavier than B by an amount  $V_{\rho g} - w$ .
  4. B is heavier than A by an amount  $V_{\rho g} - w$ .
7. If the father has blood group O and the mother has blood group AB, what are the possible blood groups of their children?
- |             |          |
|-------------|----------|
| 1. O, AB, A | 2. A, B  |
| 3. A, O     | 4. B, AB |
8. Nuclei of  $^{32}\text{P}$  and  $^{32}\text{S}$ , accelerated through the same potential difference enter a uniform, transverse magnetic field ( $Z = 15$  for P and  $Z = 16$  for S). As they emerge from the magnetic field
- |   |   |
|---|---|
| 1. both nuclei emerge undeflected                         | 2. $^{32}\text{P}$ is deflected less than $^{32}\text{S}$ |
| 3. $^{32}\text{P}$ is deflected more than $^{32}\text{S}$ | 4. both are equally deflected                             |
9. A person chewing a bubble gum did not experience ear pain in a jet plane while landing whereas another person not chewing a gum had ear pain. The reason could be
1. chewing gum is a pain killer
  2. chewing equilibrates pressure on both sides of the ear drum
  3. chewing gum closes the ear drum
  4. both are equally deflected
10. The reason why a lunar eclipse does not occur at every full moon is
1. the position of the sun is not favourable at all full moons.
  2. the orbital planes of the moon and that of the earth are inclined to each other by a small angle.
  3. the shape of the earth is not a perfect sphere.
  4. the moon reflects only from one hemisphere.



14. Water is flowing through a tube as shown. The cross-sectional areas at A and C are equal and greater than the cross-sectional area at B. If the flow is steady, then the pressure on the walls at B is



1. less than that at A and that at C
  2. more than that at A and that at C
  3. same as that at A and that at C
  4. more than that at A but less than that at C.
15. Match the two lists

Raw Material	Product
A. Limestone	a. Porcelain
B. Gypsum	b. Glass
C. Silica sand	c. Plaster of Paris
D. Clay	d. Cement

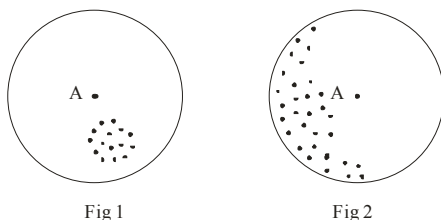
	A	B	C	D
1.	a	b	c	d
2.	d	c	b	a
3.	a	c	d	b
4.	d	a	c	b

16. The  $^{14}\text{C}$  dating method is not usually used for dating organic substances older than ~60,000 years, because
1. such objects rarely contain carbon
  2. such objects accumulated  $^{14}\text{C}$  after their formation.
  3. in those times there was no production of  $^{14}\text{C}$ .
  4. most of the  $^{14}\text{C}$  in the sample would have decayed.
17. A seismograph receives a S-wave 60 s after it receives the P-wave. If the velocities of P and S-waves are 7 km/s and 6 km/s respectively, then the distance of the seismic focus from the seismographs is
1. 2520 km
  2. 42 km
  3. 7070 km
  4. 72 km

18. The decay of a radioactive isotope P produces a stable daughter isotope D. The ratio of the number of atoms of D to the number of atoms of P after 2 half lives would be

- 1.  $1/4$
- 2.  $3/4$
- 3. 3
- 4. 2

19. The scatter plots represent the values measured by two similar instruments. Point A in the figures represents the true value. Which of the following is a correct description of the quality of these measurements?



- 1. Fig.1 : good accuracy, good precision  
Fig.2 : good accuracy, good precision
- 2. Fig.1 : poor accuracy, poor precision  
Fig.2 : good accuracy, poor precision
- 3. Fig.1 : poor accuracy, good precision  
Fig.2 : poor accuracy, poor precision
- 4. Fig.1 : poor accuracy, poor precision  
Fig.2 : poor accuracy, good precision

20. Even though the concentration of CO<sub>2</sub> is the same at sea level and at high altitude, the photosynthetic rate is higher in a plant grown at sea level than in a plant (of the same species) grown at high altitude. The reason for this is

- 1. light intensity is more at sea level.
- 2. temperature is lower at higher altitude.
- 3. atmospheric pressure is higher at sea level.
- 4. relative humidity is higher at sea level.

**PART – B**

21. A vector perpendicular to any vector that lies on the plane defined by  $x + y + z = 5$ , is

1.  $\hat{i} + \hat{j}$

2.  $\hat{j} + \hat{k}$

3.  $\hat{i} + \hat{j} + \hat{k}$

4.  $2\hat{i} + 3\hat{j} + 5\hat{k}$

22. The eigenvalues of the matrix  $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$  are

1. (1, 4, 9)

2. (0, 7, 7)

3. (0, 1, 13)

4. (0, 0, 14)

23. The first few terms in the Laurent series for  $\frac{1}{(z-1)(z-2)}$  in the region  $|z| \leq 2$ , and around  $z = 1$  is

1.  $\frac{1}{2} \left[ 1 + z + z^2 + z^3 + \dots \right] \left[ 1 + \frac{z}{2} + \frac{z^2}{4} + \frac{z^3}{8} + \dots \right]$

2.  $\frac{1}{1-z} + z - (1-z)^2 + (1-z)^3 + \dots$

3.  $\frac{1}{z^2} \left[ 1 + \frac{1}{z} + \frac{1}{z^2} + \dots \right] \left[ 1 + \frac{2}{z} + \frac{4}{z^2} + \dots \right]$

4.  $2(z-1) + 5(z-1)^2 + 7(z-1)^3 + \dots$

24. The radioactive decay of a certain material satisfies Poisson statistics with a mean rate of  $\lambda$  per second. What should be the minimum duration of counting (in seconds) so that the relative error is less than 1%?

1.  $100/\lambda$

2.  $10^4/\lambda^2$

3.  $10^4/\lambda$

4.  $1/\lambda$

25. Let  $u(x, y) = x + \frac{1}{2}(x^2 - y^2)$  be the real part of an analytic function  $f(z)$  of the complex variable  $z = x + iy$ . The imaginary part of  $f(z)$  is

- |             |                |
|-------------|----------------|
| 1. $y + xy$ | 2. $xy$        |
| 3. $y$      | 4. $y^2 - x^2$ |

26. Let  $y(x)$  be a continuous real function in the range 0 and  $2\pi$ , satisfying the inhomogeneous differential equation:

$$\sin x \frac{d^2y}{dx^2} + \cos x \frac{dy}{dx} = \delta\left(x - \frac{\pi}{2}\right).$$

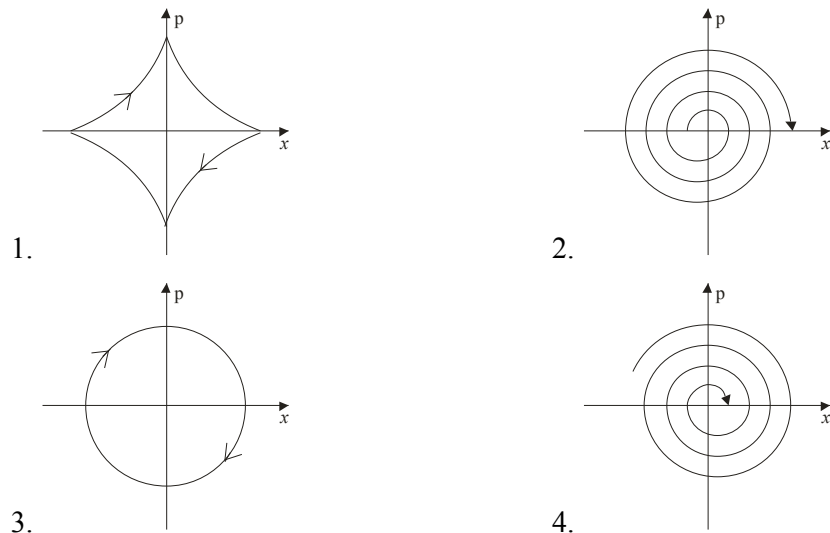
The value of  $dy/dx$  at the point  $x = \pi/2$ .

- |                               |                             |
|-------------------------------|-----------------------------|
| 1. is continuous              | 2. has a discontinuity of 3 |
| 3. has a discontinuity of 1/3 | 4. has a discontinuity of 1 |

27. A ball is picked at random from one of two boxes that contain 2 black and 3 white and 3 black and 4 white balls respectively. What is the probability that it is white?

- |          |          |
|----------|----------|
| 1. 34/70 | 2. 41/70 |
| 3. 36/70 | 4. 29/70 |

28. The bob of a simple pendulum, which undergoes small oscillations, is immersed in water. Which of the following figures best represents the phase space diagram for the pendulum?



29. Two events, separated by a (spatial) distance  $9 \times 10^9 \text{ m}$ , are simultaneous in one inertial frame. The time interval between these two events in a frame moving with a constant speed  $0.8 c$  (where the speed of light  $c = 3 \times 10^8 \text{ m/s}$ ) is

- |         |         |
|---------|---------|
| 1. 60 s | 2. 40 s |
| 3. 20s  | 4. 0 s  |

30. If the Lagrangian of particle moving in one dimensions is given by  $L = \frac{\dot{x}^2}{2x} - V(x)$ , the

Hamiltonian is

- |                                     |                                  |
|-------------------------------------|----------------------------------|
| 1. $\frac{1}{2} x p^2 + V(x)$       | 2. $\frac{\dot{x}^2}{2x} + V(x)$ |
| 3. $\frac{1}{2} \tilde{x}^2 + V(x)$ | 4. $\frac{p^2}{2x} + V(x)$       |

31. A horizontal circular platform rotates with a constant angular velocity  $\Omega$  directed vertically upwards. A person seated at the centre shoots a bullet of mass  $m$  horizontally with speed  $v$ . The acceleration of the bullet in the reference frame of the shooter is

- |                            |                           |
|----------------------------|---------------------------|
| 1. $2v\Omega$ to his right | 2. $2v\Omega$ to his left |
| 3. $v\Omega$ to his right  | 4. $v\Omega$ to his left  |

32. The magnetic field corresponding to the vector potential

$$\vec{A} = \frac{1}{2} \vec{F} \times \vec{r} + \frac{10}{r^3} \vec{r}$$

where  $\vec{F}$  is a constant vector, is

- |                                       |                                       |
|---------------------------------------|---------------------------------------|
| 1. $\vec{F}$                          | 2. $-\vec{F}$                         |
| 3. $\vec{F} + \frac{30}{r^4} \vec{r}$ | 4. $\vec{F} - \frac{30}{r^4} \vec{r}$ |

33. An electromagnetic wave is incident on a water-air interface. The phase of the perpendicular component of the electric field,  $E_{\perp}$ , of the reflected wave into the water is found to remain the same for all angles of incidence. The phases of the magnetic field H

- |                       |                        |
|-----------------------|------------------------|
| 1. does not change    | 2. changes by $3\pi/2$ |
| 3. changes by $\pi/2$ | 4. changes by $\pi$    |

34. The magnetic field at a distance  $R$  from a long straight wire carrying a steady current  $I$  is proportional to

- |              |            |
|--------------|------------|
| 1. $IR$      | 2. $I/R^2$ |
| 3. $I^2/R^3$ | 3. $I/R$   |

35. The component along an arbitrary direction  $\hat{n}$ , with direction cosines  $(n_x, n_y, n_z)$ , of the spin of a spin  $-\frac{1}{2}$  particle is measured. The result is

- |  |                              |
|--|------------------------------|
| 1. 0                                       | 2. $\pm \frac{\hbar}{2} n_z$ |
| 3. $\pm \frac{\hbar}{2} (n_x + n_y + n_z)$ | 4. $\pm \frac{\hbar}{2}$     |

36. A particle of mass  $m$  is in cubic box of size  $a$ . The potential inside the box ( $0 \leq x < a, 0 \leq y < a, 0 \leq z < a$ ) is zero and infinite outside. If the particle is in an eigenstate of energy  $E = \frac{14\pi^2\hbar^2}{2ma}$ , its wavefunctions is

- |  |  |
|--|--|
| 1. $\psi = \left(\frac{2}{a}\right)^{3/2} \sin \frac{3\pi x}{a} \sin \frac{5\pi y}{a} \sin \frac{6\pi z}{a}$ | 2. $\psi = \left(\frac{2}{a}\right)^{3/2} \sin \frac{7\pi x}{a} \sin \frac{4\pi y}{a} \sin \frac{3\pi z}{a}$ |
| 3. $\psi = \left(\frac{2}{a}\right)^{3/2} \sin \frac{4\pi x}{a} \sin \frac{8\pi y}{a} \sin \frac{2\pi z}{a}$ | 4. $\psi = \left(\frac{2}{a}\right)^{3/2} \sin \frac{\pi x}{a} \sin \frac{2\pi y}{a} \sin \frac{3\pi z}{a}$  |

37. Let  $\psi_{nlm}$  denote the eigenfunctions of a Hamiltonian for a spherically symmetric

potential  $V(r)$ . The wavefunction  $\psi = \frac{1}{4} [\psi_{210} + \sqrt{5}\psi_{21-1} + \sqrt{10}\psi_{211}]$  is an eigenfunction

only of

- |                       |                    |
|-----------------------|--------------------|
| 1. $H, L^2$ and $L_z$ | 2. $H$ and $L_z$   |
| 3. $H$ and $L^2$      | 4. $L^2$ and $L_z$ |

38. The commutator  $[x^2, p^2]$  is

- |             |                    |
|-------------|--------------------|
| 1. $2ih xp$ | 2. $2ih (xp + px)$ |
| 3. $2ih px$ | 4. $2ih (xp - px)$ |

39. Consider a system of non-interacting particles in  $d$ -dimensions obeying the dispersion relation  $\varepsilon = Ak^s$ , where  $\varepsilon$  is the energy,  $k$  is the wave vector,  $s$  is an integer and  $A$  a constant. The density of states  $N(\varepsilon)$ , is proportional to

- |                                  |                                  |
|----------------------------------|----------------------------------|
| 1. $\varepsilon^{\frac{1}{d}-1}$ | 2. $\varepsilon^{\frac{d}{1}-1}$ |
| 3. $\varepsilon^{\frac{d}{3}-1}$ | 4. $\varepsilon^{\frac{3}{d}-1}$ |

40. The number of ways in which  $N$  identical bosons can be distributed in two energy levels, is

- |                 |                 |
|-----------------|-----------------|
| 1. $N + 1$      | 2. $N(N - 1)/2$ |
| 3. $N(N + 1)/2$ | 4. $N$          |

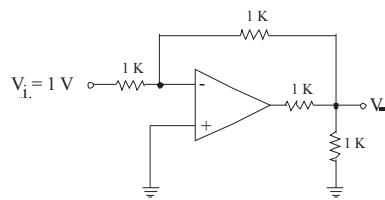
41. The free energy of a gas of  $N$  particles in a volume  $V$  and at a temperature  $T$  is

$$F = Nk_B T \ln \left[ a_0 V (k_B T)^{5/2} / N \right],$$

where  $a_0$  is a constant and  $k_B$  denotes the Boltzmann constant. The internal energy of the gas is

- |   |  |
|---|--|
| 1. $\frac{3}{2} Nk_B T$   | 2. $\frac{5}{2} Nk_B T$                              |
| 3. $Nk_B T \ln \left[ a_0 V (k_B T)^{5/2} / N \right] - \frac{3}{2} Nk_B T$ | 4. $Nk_B T \ln \left[ a_0 V / (k_B T)^{5/2} \right]$ |

42. In the op-amp circuit shown in the figure below, the input voltage  $V_i$  is  $1V$ . The value of the output  $V_o$  is

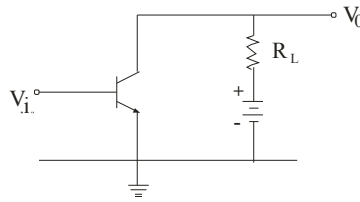


- |             |             |
|-------------|-------------|
| 1. $-0.33V$ | 2. $-0.50V$ |
| 3. $-1.00V$ | 4. $-0.25V$ |

43. An LED operates at 1.5 V and 5 mA in forward bias. Assuming an 80% external efficiency of the LED, how many photons are emitted per second?

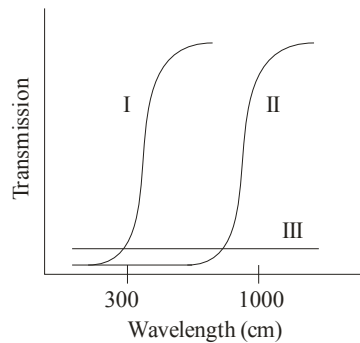
- |                         |                         |
|-------------------------|-------------------------|
| 1. $5.0 \times 10^{16}$ | 2. $1.5 \times 10^{16}$ |
| 3. $0.8 \times 10^{16}$ | 4. $2.5 \times 10^{16}$ |

44. The transistor in the given circuit has  $h_{fe} = 35\Omega$  and  $h_{ie} = 1000\Omega$ . If the load resistance  $R_L = 1000\Omega$ , the voltage and current gain are, respectively.



- |                  |                  |
|------------------|------------------|
| 1. - 35 and +35  | 2. 35 and - 35   |
| 3. 35 and - 0.97 | 4. 0.98 and - 35 |

45. The experimentally measured transmission spectra of metal, insulator and semiconductor thin films are shown in the figure. It can be inferred that I, II and III correspond, respectively, to



- |                                       |                                       |
|---------------------------------------|---------------------------------------|
| a) insulator, semiconductor and metal | b) semiconductor, metal and insulator |
| c) metal, semiconductor and insulator | d) insulator, metal and semiconductor |

## PART – C

46. The eigenvalues of the antisymmetric matrix,  $A = \begin{pmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_3 & n_1 & 0 \end{pmatrix}$  where

$n_1, n_2$  and  $n_3$  are the components of a unit vector, are

- |                      |               |
|----------------------|---------------|
| 1. 0, $i$ , $-i$     | 2. 0, 1, $-1$ |
| 3. 0, $1+i$ , $-1-i$ | 4. 0, 0, 0    |

47. Which of the following limits exists?

- |   |  |
|---|--|
| 1. $\lim_{N \rightarrow \infty} \left( \sum_{n=1}^N \frac{1}{m} + \ln N \right)$        | 2. $\lim_{N \rightarrow \infty} \left( \sum_{n=1}^N \frac{1}{m} - \ln N \right)$ |
| 3. $\lim_{N \rightarrow \infty} \left( \sum_{n=1}^N \frac{1}{\sqrt{m}} - \ln N \right)$ | 4. $\lim_{N \rightarrow \infty} \sum_{n=1}^N \frac{1}{m}$                        |

48. A bag contains many balls, each with a number painted on it. There are exactly  $n$  balls which have the number  $n$  (namely one ball with 1, two balls with 2, and so on until  $N$  balls with  $N$  on them). An experiment consists of choosing a ball at random, noting the number on it and returning it to the bag. If the experiment is represented a large number of times, the average value of the number will tend to

- |                     |                       |
|---------------------|-----------------------|
| 1. $\frac{2N+1}{3}$ | 2. $\frac{N}{2}$      |
| 3. $\frac{N+1}{2}$  | 4. $\frac{N(N+1)}{2}$ |

49. The value of the internal  $\int_{-\infty}^{\infty} \frac{1}{t^2 - R^2} \cos\left(\frac{rt}{2R}\right) dt$  is

- |              |             |
|--------------|-------------|
| 1. $-2\pi/R$ | 2. $-\pi/R$ |
| 3. $\pi/R$   | 4. $2\pi/R$ |

50. The Poisson bracket  $\{\mathbf{r}, |\mathbf{p}|\}$  has the value

- |                               |                       |
|-------------------------------|-----------------------|
| 1. $ \mathbf{r}  \mathbf{p} $ | 2. $\bar{r}, \bar{p}$ |
| 3. 3                          | 4. 1                  |

51. Consider the motion of a classical particle in a one dimensional double-well potential

$$V(x) = \frac{1}{4}(x^2 - 2).$$

If the particle is displaced infinitesimally from the minimum on the positive x-axis (and friction is neglected), then

- the particle will execute simple harmonic motion in the right well with an angular frequency  $\omega = \sqrt{2}$
- the particle will execute simple harmonic motion in the right well with an angular frequency  $\omega = 2$
- the particle will switch between the right and left wells
- the particle will approach the bottom of the right well and settle there

52. What is the proper time interval between the occurrence of two events if in one inertial frame the events are separated by  $7.5 \times 10^8$  m and occur 6.5s apart?

- |          |          |
|----------|----------|
| 1. 6.50s | 2. 6.00s |
| 3. 5.75s | 4. 5.00s |

53. A free particle described by a plane wave and moving in the positive z-direction undergoes scattering by a potential

$$V(r) = \begin{cases} V_0 & \text{if } r \leq R \\ 0 & \text{if } r > R \end{cases}$$

If  $V_0$  is changed to  $2V_0$ , keeping R fixed, then the differential scattering cross-section in the Born approximation.

- |   |   |
|---|---|
| a) increases to four times the original value | b) increases to twice the original value      |
| c) decreases to half the original value       | d) decreases to one fourth the original value |

54. A variational calculation is done with the normalized trial wavefunction

$$\psi(x) = \frac{\sqrt{15}}{4a^{5/2}}(a^2 - x^2) \text{ for the one dimensional potential well}$$

$$V(x) = \begin{cases} 0 & \text{if } |x| \leq a \\ \infty & \text{if } |x| \geq a \end{cases}$$

The general state energy is estimated to be

- |                         |                         |
|-------------------------|-------------------------|
| 1. $\frac{5h^2}{3ma^2}$ | 2. $\frac{3h^2}{2ma^2}$ |
| 3. $\frac{3h^2}{5ma^2}$ | 4. $\frac{5h^2}{4ma^2}$ |

55. A particle in one-dimension is in the potential

$$V(x) = \begin{cases} \infty & \text{if } x < 0 \\ -V & \text{if } 0 \leq x \leq t \\ 0 & \text{if } x > t \end{cases}$$

If these as at least one bound state, the minimum depth of the potential is

- |                                  |                                  |
|----------------------------------|----------------------------------|
| 1. $\frac{\hbar^2 \pi^2}{8ml^2}$ | 2. $\frac{\hbar^2 \pi^2}{2ml^2}$ |
| 3. $\frac{2\hbar^2 \pi^2}{ml^2}$ | 4. $\frac{\hbar^2 \pi^2}{ml^2}$  |

56. Which of the following is a self-adjoint operator in the spherical polar coordinate system

$(r, \theta, \phi)$ ?

- |   |   |
|---|---|
| 1. $-\frac{i\hbar}{\sin^2 \theta} \frac{\partial}{\partial \theta}$ | 2. $-i\hbar \frac{\partial}{\partial \theta}$             |
| 5. $-\frac{i\hbar}{\sin \theta} \frac{\partial}{\partial \theta}$   | 4. $-i\hbar \sin \theta \frac{\partial}{\partial \theta}$ |

57. Which of the following quantities is Lorentz invariant?

- |                     |                    |
|---------------------|--------------------|
| 1. $ E \times B ^2$ | 2. $ E ^2 -  B ^2$ |
| 3. $ E ^2 +  B ^2$  | 4. $ E ^2  B ^2$   |



61. The magnetization  $M$  of a ferromagnet, as a function of the temperature  $T$  and the magnetic field  $H$ , is described by the equation  $M = \tanh\left(\frac{T_c}{T}M + \frac{H}{T}\right)$ . In these units, the zero-field magnetic susceptibility in terms of  $M(0) = M(H = 0)$  is given by

- |   |                                 |
|---|---------------------------------|
| 1. $\frac{1 - M^2(0)}{T - T_c(1 - M^2(0))}$ | 2. $\frac{1 - M^2(0)}{T - T_c}$ |
| 3. $\frac{1 - M^2(0)}{T + T_c}$             | 4. $\frac{1 - M^2(0)}{T}$       |

62. Bose condensation occurs in liquid  $He^4$  kept at ambient pressure at 2.17 K. At which temperature will Bose condensation occur in  $He^4$  in gaseous state, the density of which is 1000 times smaller than that of liquid  $He^4$ ? (Assume that it is a perfect Bose gas.)

- |                 |                 |
|-----------------|-----------------|
| 1. 2.17 mK      | 2. 21.7 mK      |
| 3. 21.7 $\mu$ K | 4. 2.17 $\mu$ K |

63. Consider black body radiation contained in a cavity whose walls are at temperature  $T$ . The radiation is in equilibrium with the walls of the cavity. If the temperature of the walls is increased to  $2T$  and the radiation is allowed to come to equilibrium at the new temperature, the entropy of the radiation increases by a factor of

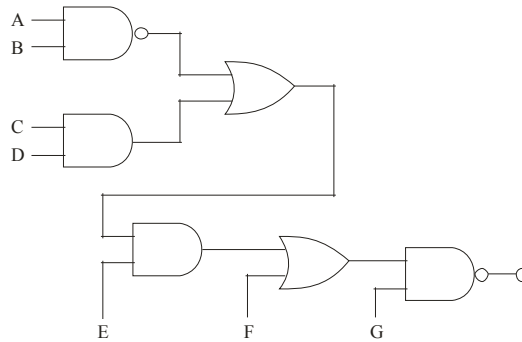
- |      |       |
|------|-------|
| 1. 2 | 2. 4  |
| 3. 8 | 4. 16 |

64. The output  $O$  of the given circuit cases I and II, where

Case I:  $A, B = 1; C, D = 0; E, F = 1$  and  $G = 0$

Case II:  $A, B = 0; C, D = 0; E, F = 0$  and  $G = 1$

are, respectively



- |        |        |
|--------|--------|
| 1. 1,0 | 2. 0,1 |
| 3. 0,0 | 4. 1,1 |

65. A resistance strain gauge is fastened to a steel fixture and subjected to a stress of  $1000 \text{ kg/m}^2$ . If the gauge factor is 3 and the modulus of elasticity of steel is  $2 \times 10^{10} \text{ kg/m}^2$ , then the fractional change in resistance of the strain gauge due to the applied stress is

(Note: The gauge factor is defined as the ratio of the fractional change in resistance to the fractional change in length).

- |                           |                         |
|---------------------------|-------------------------|
| 1. $1.5 \times 10^{-7}$   | 2. $3.0 \times 10^{-7}$ |
| 3. $0.16 \times 10^{-10}$ | 4. $0.5 \times 10^{-7}$ |

66. Consider a sinusoidal waveform of amplitude  $1V$  and frequency  $f_0$ . Starting from an arbitrary initial time, the wavefunction is sampled at intervals of  $1/(2f_0)$ . If the corresponding Fourier spectrum peaks at a frequency  $\bar{f}$  and an amplitude  $\bar{A}$ , then

- |  |  |
|--|--|
| 1. $\bar{f} = 2f_0$ and $\bar{A} = 1V$ | 2. $\bar{f} = f_0$ and $0 \leq \bar{A} \leq 1V$                  |
| 3. $\bar{f} = 0$ and $\bar{A} = 1V$    | 4. $\bar{f} = \frac{f_0}{2}$ and $\bar{A} = \frac{1}{\sqrt{2}}V$ |

67. The first absorption spectrum of  $^{12}\text{C}^{16}\text{O}$  is at  $3.842\text{ cm}^{-1}$  while that of  $^{13}\text{C}^{16}\text{O}$  is at  $3.673\text{ cm}^{-1}$ . The ratio of their moments of inertia is

- |          |          |
|----------|----------|
| 1. 1.851 | 2. 1.286 |
| 3. 1.046 | 4. 1.038 |

68. The spin-orbit interaction in an atom is given by  $H = aL.S$ , where L and S denote the orbital and spin angular momenta, respectively, of the electron. The splitting between the levels  $^2P_{3/2}$  and  $^2P_{1/2}$  is

- |                          |                          |
|--------------------------|--------------------------|
| 1. $\frac{3}{2}a\hbar^2$ | 2. $\frac{1}{2}a\hbar^2$ |
| 3. $3a\hbar^2$           | 4. $\frac{5}{2}a\hbar^2$ |

69. The spectral line corresponding to an atomic transition from  $J = 1$  to  $J = 0$  states splits in a magnetic field of 1kG into three components separated by  $1.6 \times 10^{-5}\text{ \AA}$ . If the zero field spectral line corresponds to  $1849\text{ \AA}$ , what is the g-factor corresponding to the  $J = 1$  state?

(You may use  $\frac{hc}{\mu_0} = 2 \times 10^4\text{ cm}$ .)

- |      |        |
|------|--------|
| 1. 2 | 2. 3/2 |
| 3. 1 | 4. 1/2 |

70. The energy required to create a lattice vacancy in a crystal is equal to 1 eV. The ratio of the number densities of vacancies  $n(1200\text{K})/n(300\text{K})$ , when the crystal is at equilibrium at 1200 K and 300 K, respectively, is approximately

- |                |                |
|----------------|----------------|
| 1. $\exp(-30)$ | 2. $\exp(-15)$ |
| 3. $\exp(15)$  | 4. $\exp(30)$  |

71. The dispersion relation of photons in a solid is given by  $\omega^2(k) = \omega_0^2(3 - \cos k_1 a - \cos k_2 a - \cos k_3 a)$ . The velocity of the phonons at large wavelength is

- |                            |                            |
|----------------------------|----------------------------|
| 1. $\omega_0 a / \sqrt{3}$ | 2. $\omega_0 a$            |
| 3. $\sqrt{3}\omega_0 a$    | 4. $\omega_0 a / \sqrt{2}$ |

72. Consider an electron in a box of length  $L$  with periodic boundary condition

$\psi(x) = \psi(x + L)$ . If the electron is in the  $\psi_k(x) = \frac{1}{\sqrt{L}} e^{ikx}$  with energy  $\varepsilon_k = \frac{\hbar^2 k^2}{2m}$ , what is

the correction to its energy, to second order of perturbation theory, when it is subjected to a weak periodic potential  $V(x) = V_0 \cos gx$  where  $g$  is an integral multiple of the  $2\pi/L$ ?

- |  |  |
|--|--|
| 1. $V_0^2 \varepsilon_g / \varepsilon_k^2$                   | 2. $-\frac{mV_0^2}{2\hbar^2} \left( \frac{1}{g^2 + 2kg} + \frac{1}{g^2 - 2kg} \right)$ |
| 3. $V_0^2 (\varepsilon_k - \varepsilon_g) / \varepsilon_g^2$ | 4. $V_0^2 / (\varepsilon_k + \varepsilon_g)$   |

73. The ground state of  ${}^{207}_{82}\text{Pb}$  nucleus has spin-parity  $J^P = \frac{1}{2}^-$ , while the first excited state

has  $J^P = \frac{5}{2}^-$ . The electromagnetic radiation emitted when the nucleus makes a transition

from the first excited state to the ground state are

- |              |              |
|--------------|--------------|
| 1. E2 and E3 | 2. M2 and E3 |
| 3. E2 and M3 | 4. M2 and M3 |

74. The dominant interactions underlying the following processes

A.  $K^- + p \rightarrow \Sigma + \pi$

B.  $\mu^- + \mu^+ \rightarrow K^- + K^+$

C.  $\Sigma' \rightarrow p + \pi^a$  are

1. A: strong, B: electromagnetic and C: weak
2. A: strong, B: weak and C: weak
3. A: weak, B: electromagnetic and C: strong
4. A: weak, B: electromagnetic and C: weak

75. If a Higgs boson of mass  $m_H$  moving with a speed  $\beta = \frac{v}{c}$  decays into a pair of photons,

then the invariant mass of the photon pair is

[Note: The invariant mass of a system of two particles, with four-momenta  $p_1$  and  $p_2$  is

$$(p_1 + p_2)^2]$$

1.  $\beta m_H$

2.  $m_H$

3.  $m_H / \sqrt{1 - \beta^2}$

4.  $\beta m_H / \sqrt{1 - \beta^2}$